

Problem #1 (part 1)

Student# = 312

Let's first assign a variable to the amount of trees the farmer has.

• amount of trees = x , where x is a whole number.

Now let's write 2 equations using x .

$$x + 20 = y^2 \quad x - 39 = z^2$$

y and z are both whole numbers.

Now we can subtract these 2 equations to get

$$\begin{array}{r} x + 20 = y^2 \\ -(x - 39 = z^2) \\ \hline 59 = y^2 - z^2 \end{array} \rightarrow 59 + z^2 = y^2$$

Since we know they are both whole numbers, we know that they have a whole number difference. We also know that y is greater than z , so we can rewrite y as $z + a$, where a is a whole number. Now we can plug this into the equation:

$$59 + z^2 = (z + a)^2$$

this boils down to...

$$59 + z^2 = z^2 + 2za + a^2$$

$$59 = 2za + a^2$$

Now we just have to find a solution where a and z are both whole numbers.

We can rewrite this as $59 = a(2z + a)$

Now since we know that both a and z must be whole numbers, and 59 only has factors 1 and 59, we know that $a = 1$ because a cannot be 59, otherwise z is negative.

Now with a being 1 we rewrite the equation as:

$$59 = 2z + 1$$

$$58 = 2z \quad z = 29$$

where $z = 29$

✓ (29)